

THE INFINITY MIRROR TEST FOR GRAPH GENERATORS

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University of Notre Dame

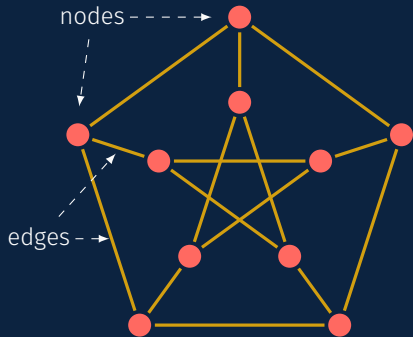
July 9, 2020





Preliminaries

WHAT ARE GRAPHS?



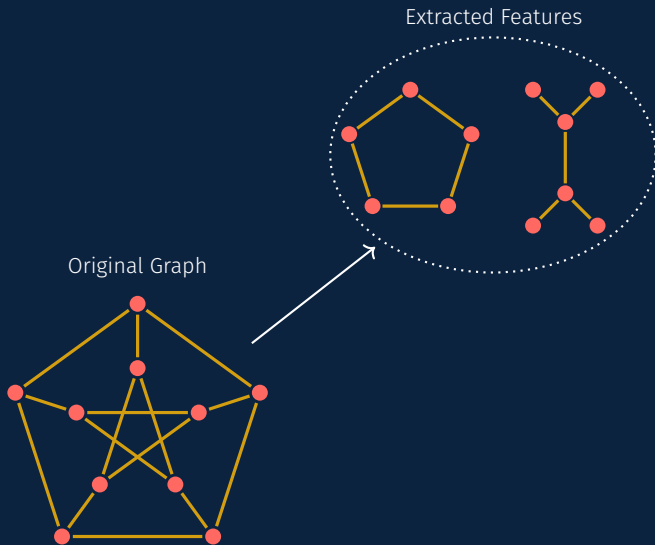
Domain	Nodes	Edges
<i>World Wide Web</i>	Webpages	Hyperlinks
<i>Scientific Papers</i>	Papers	Citations
<i>Flights</i>	Airports	Non-stop flights
<i>Facebook</i>	Users	Friendships

WHAT ARE GRAPH GENERATIVE MODELS?

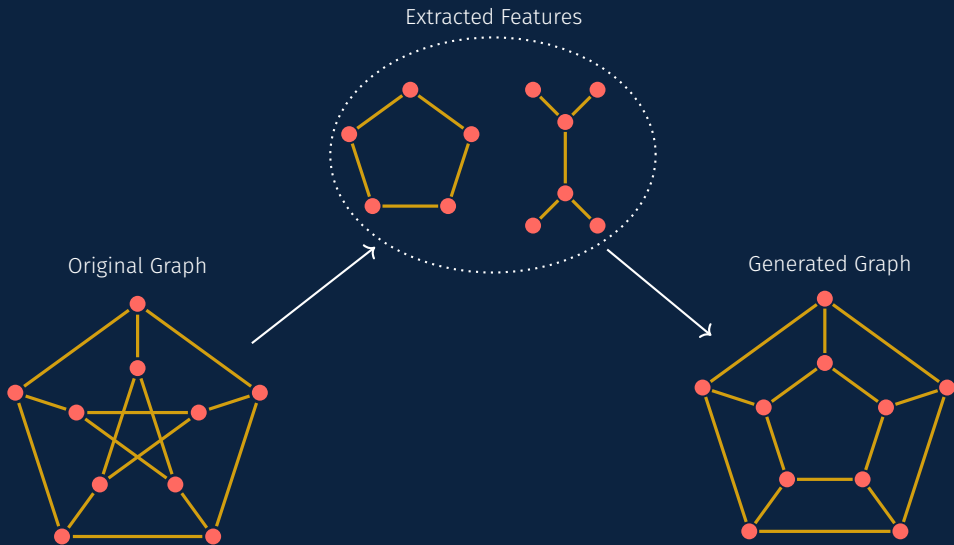
Original Graph



WHAT ARE GRAPH GENERATIVE MODELS?



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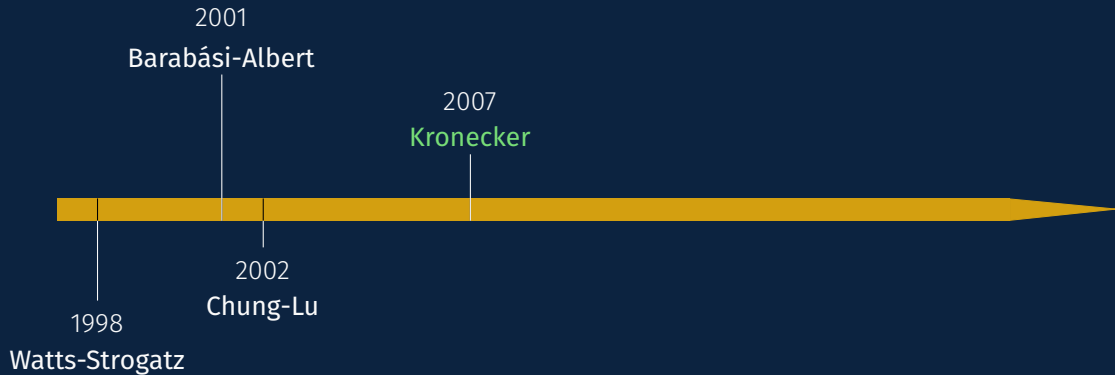
A BRIEF HISTORY OF RECENT GRAPH GENERATORS



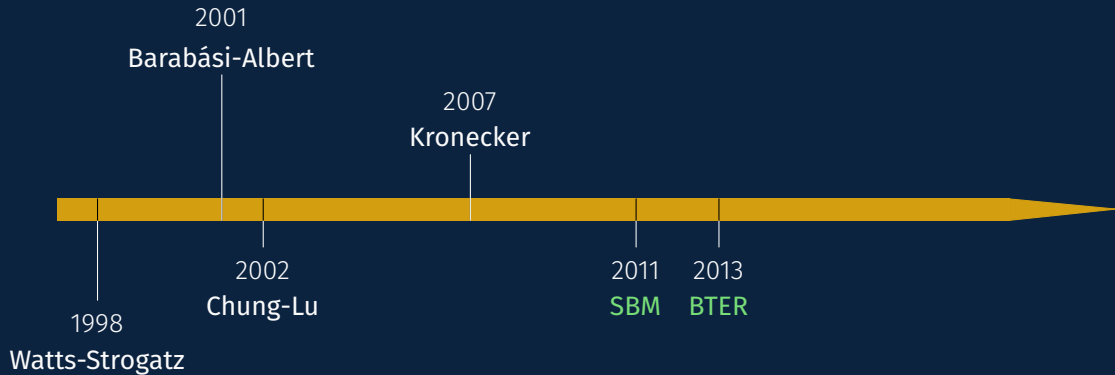
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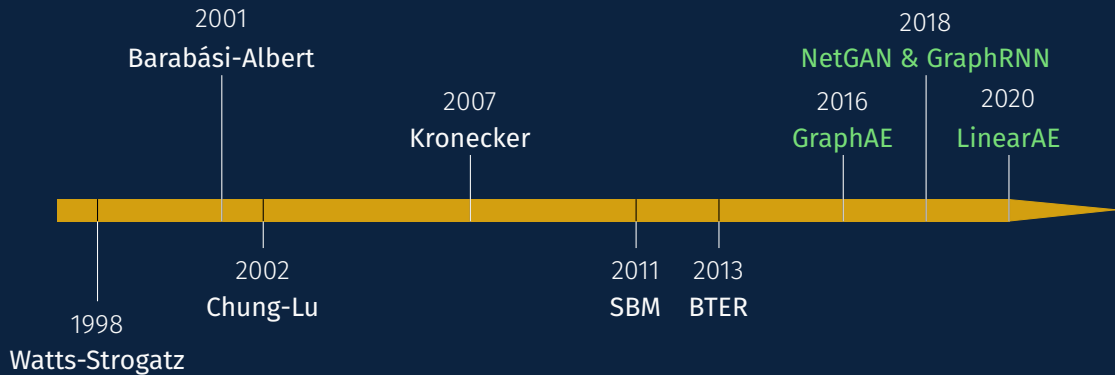
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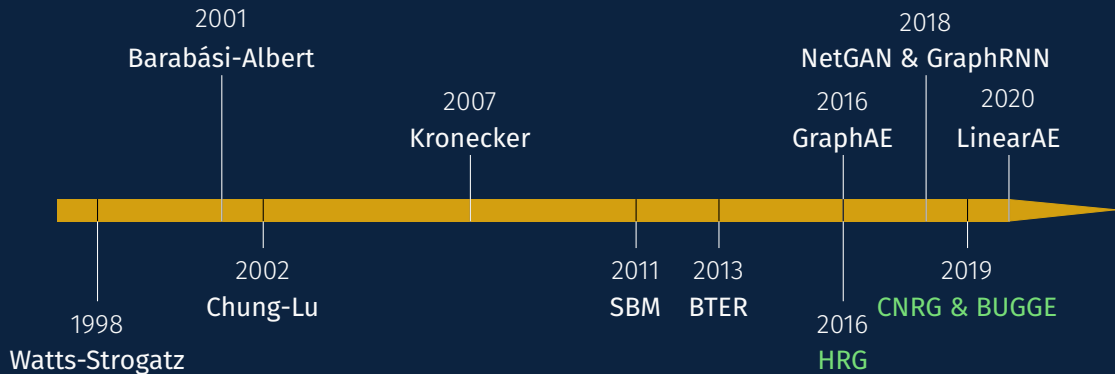
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An aerial photograph of Prague, Czech Republic, taken during the "blue hour" of sunset. The Vltava River flows through the center of the city. In the foreground, the green copper dome of the Old Town Hall is visible on the left, and another similar dome is on the right. The Charles Bridge, a long stone bridge with many arches, spans the river in the middle ground. The city's dense architecture, with red-tiled roofs and various church spires, is visible in the background under a soft, hazy sky. The title "The Infinity Mirror Test" is centered in white text over the river and bridge area.

The Infinity Mirror Test

THE INFINITY MIRROR TEST FOR DUCK VADER



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THE INFINITY MIRROR TEST: A STRESS TEST FOR GRAPH GENERATORS

Key Idea

Iteratively fitting and generating from a model will **amplify** the model's implicit biases.



Figure 1: The Infinity Mirror Test

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Methodology

Given an initial graph G_0 and a model $\mathcal{M}$, compute 50 **independent** chains of graphs $\langle G_1, G_2, \dots, G_n \rangle$.

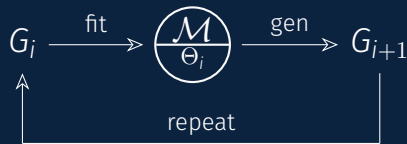


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Evaluation

Compare graph G_i with G_0 in a given chain to expose different biases.

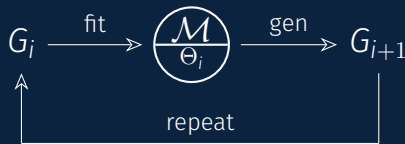
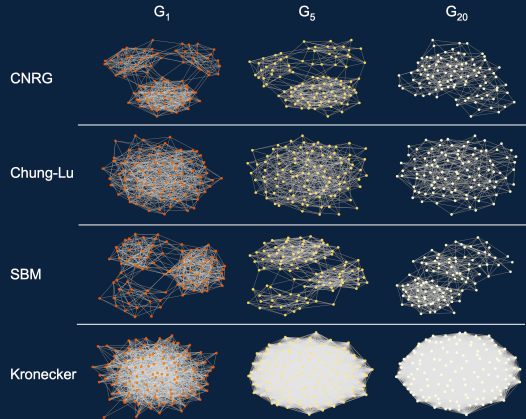


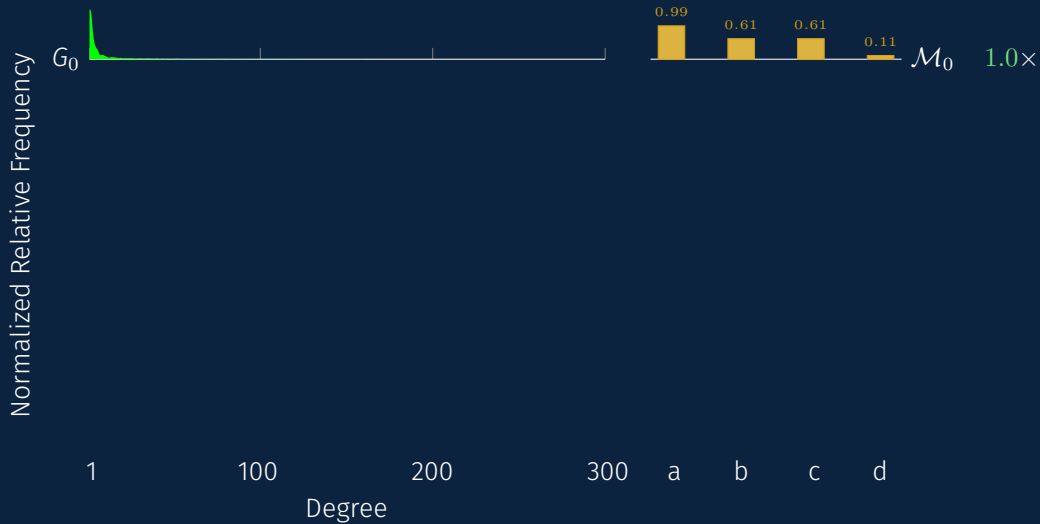
Figure 1: The Infinity Mirror Test

EXAMPLE INFINITY MIRROR



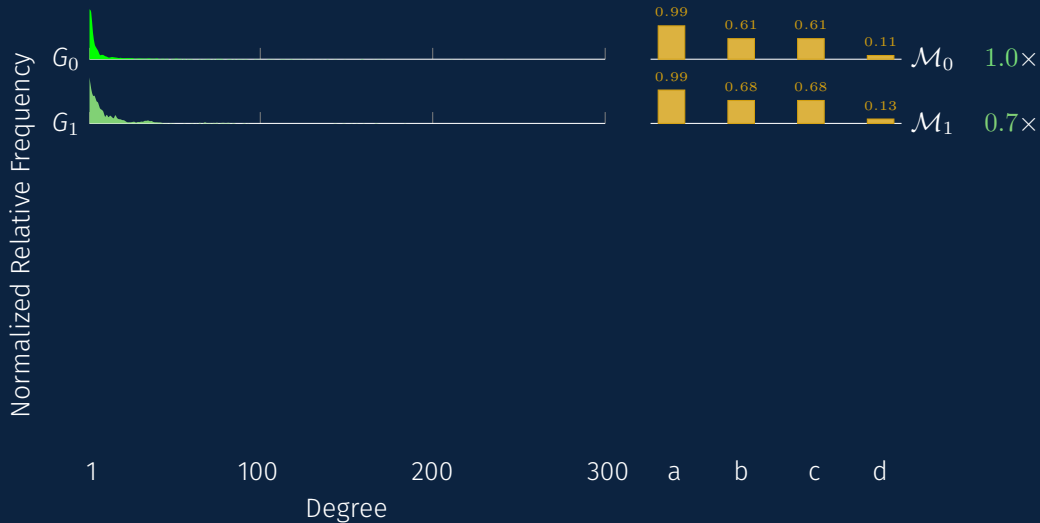
KRONECKER DEGREE DISTRIBUTION RIDGE PLOT

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



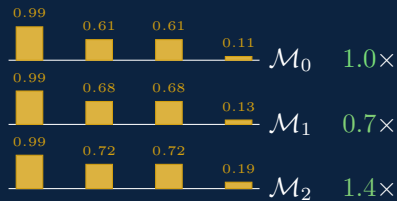
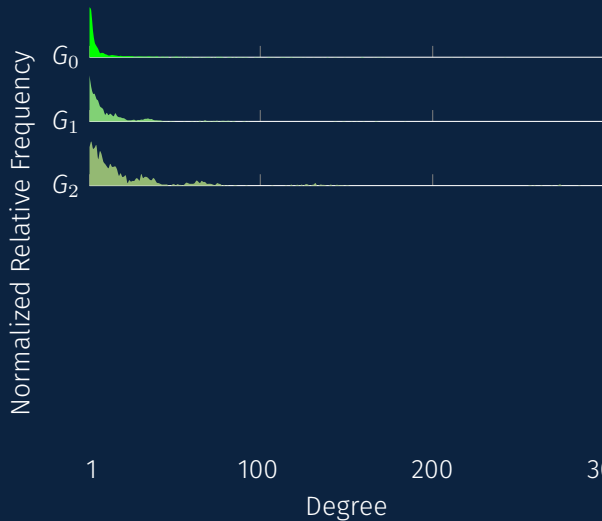
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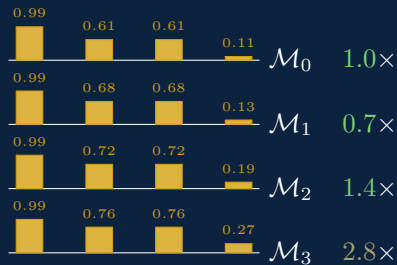
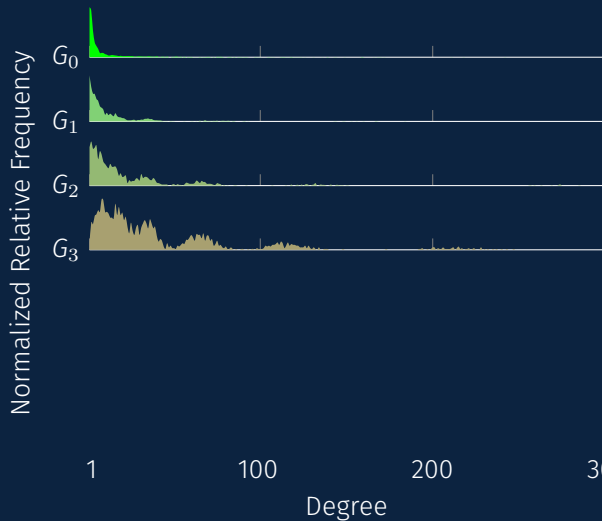
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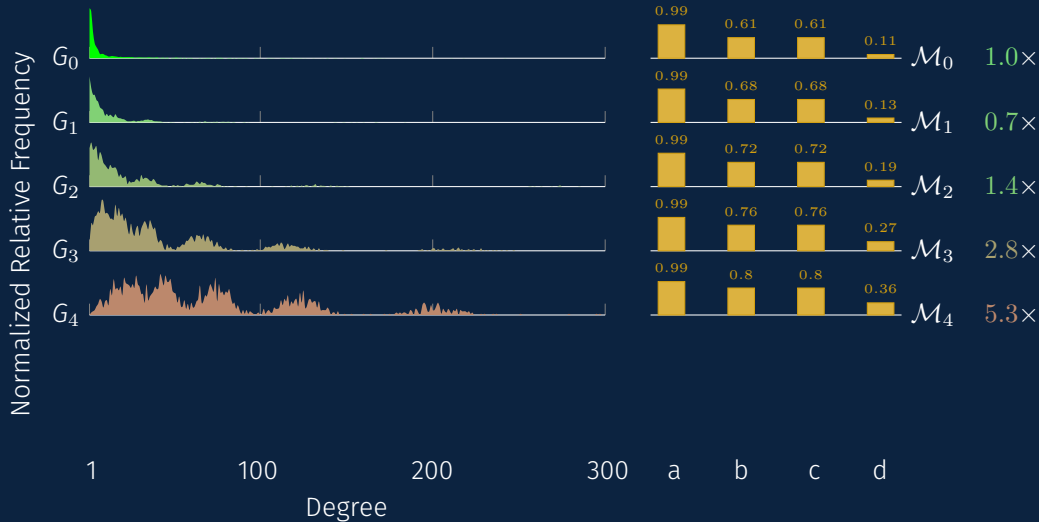
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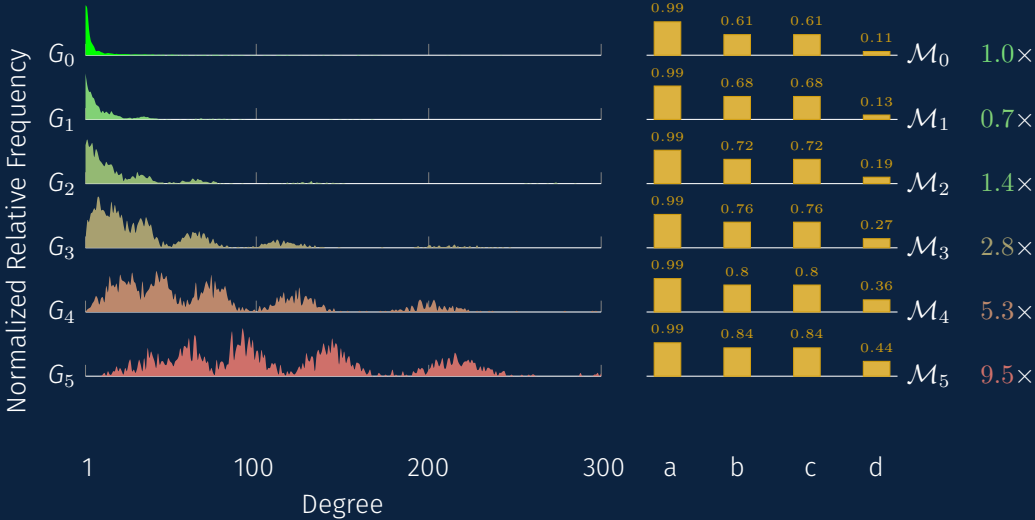
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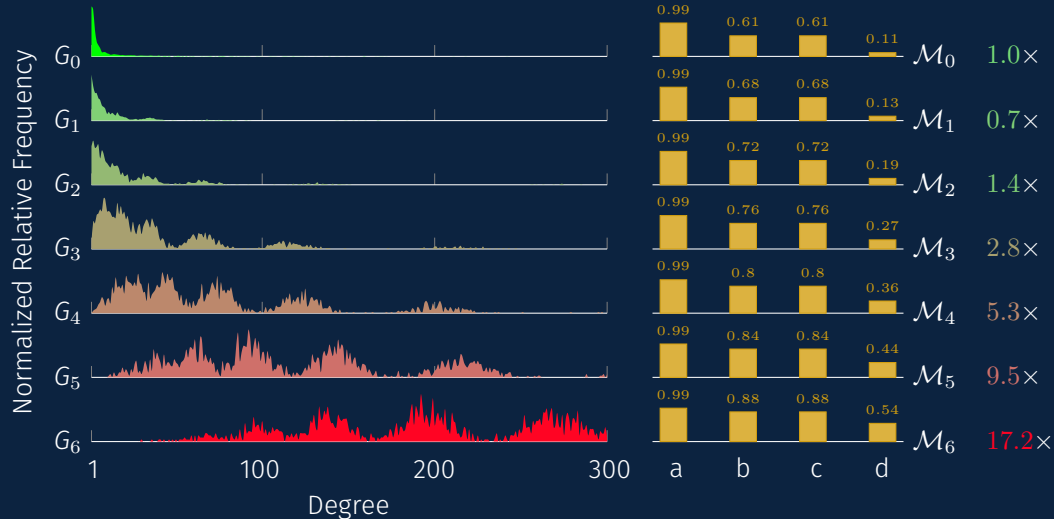
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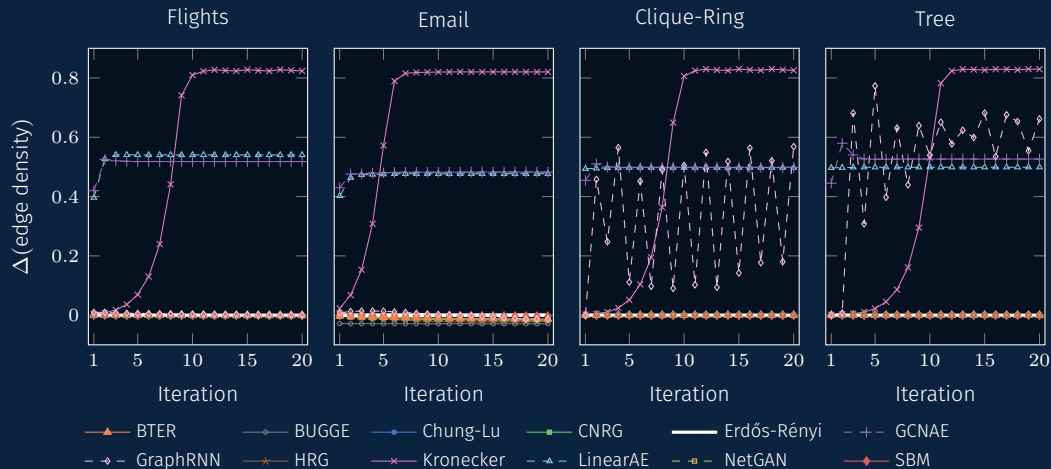


KRONECKER DEGREE DISTRIBUTION RIDGE PLOT

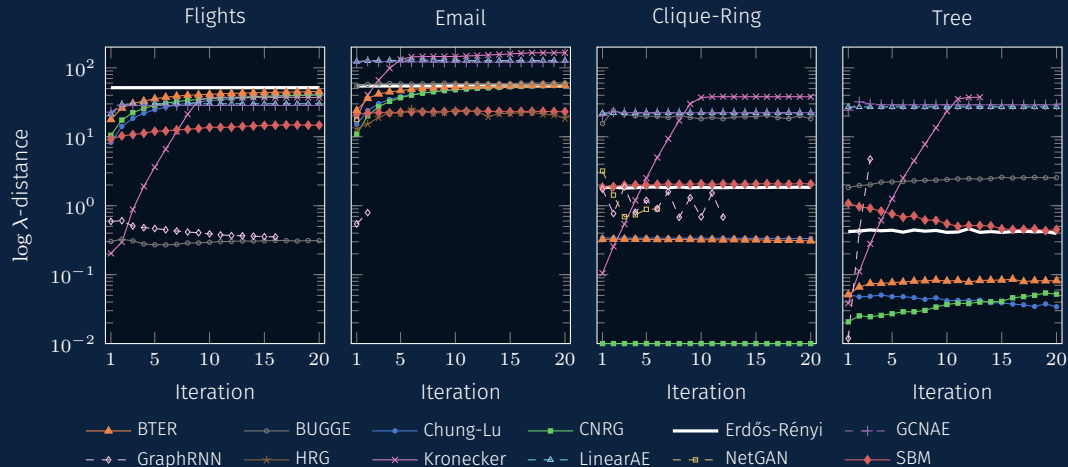
$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



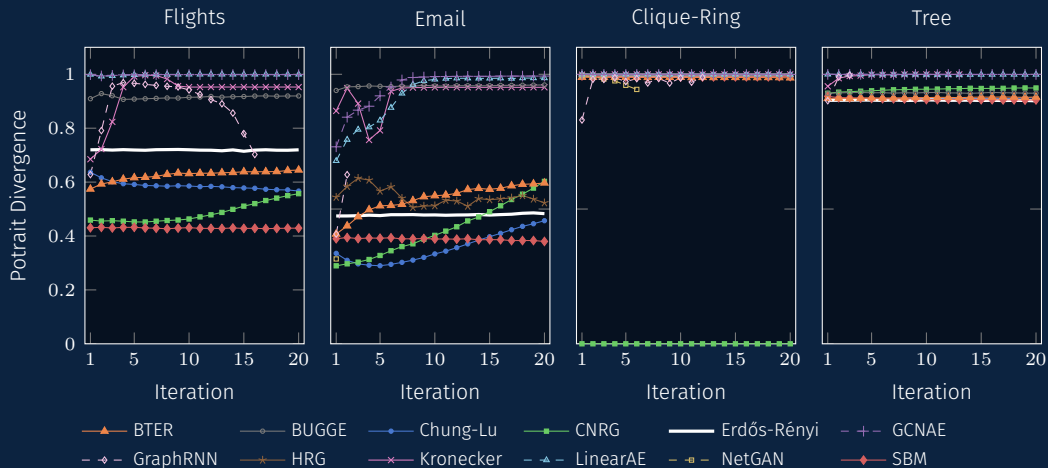
EVOLUTION OF RELATIVE EDGE DENSITY ACROSS ITERATIONS



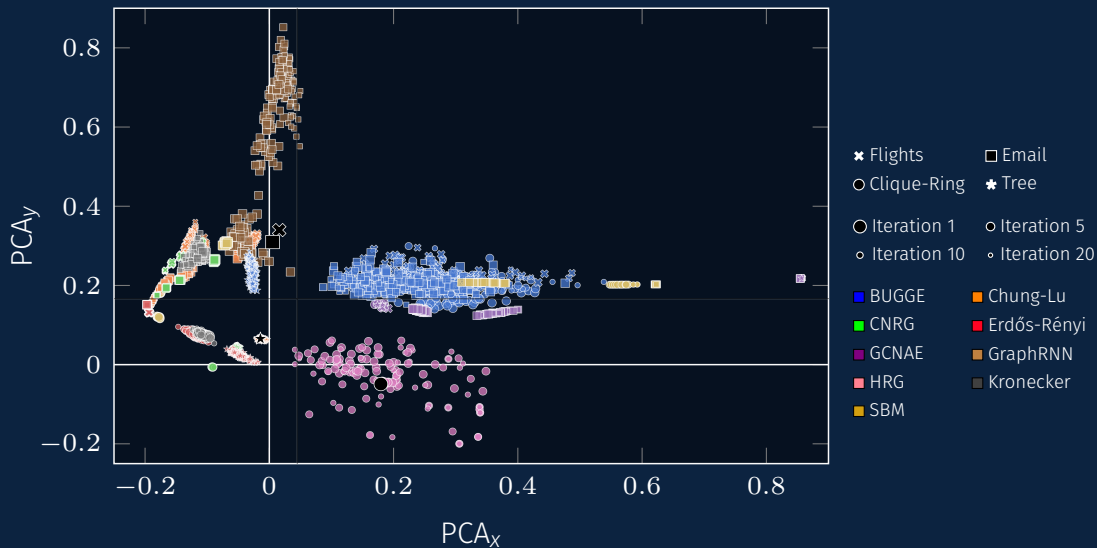
EVOLUTION OF λ -DISTANCE ACROSS ITERATIONS



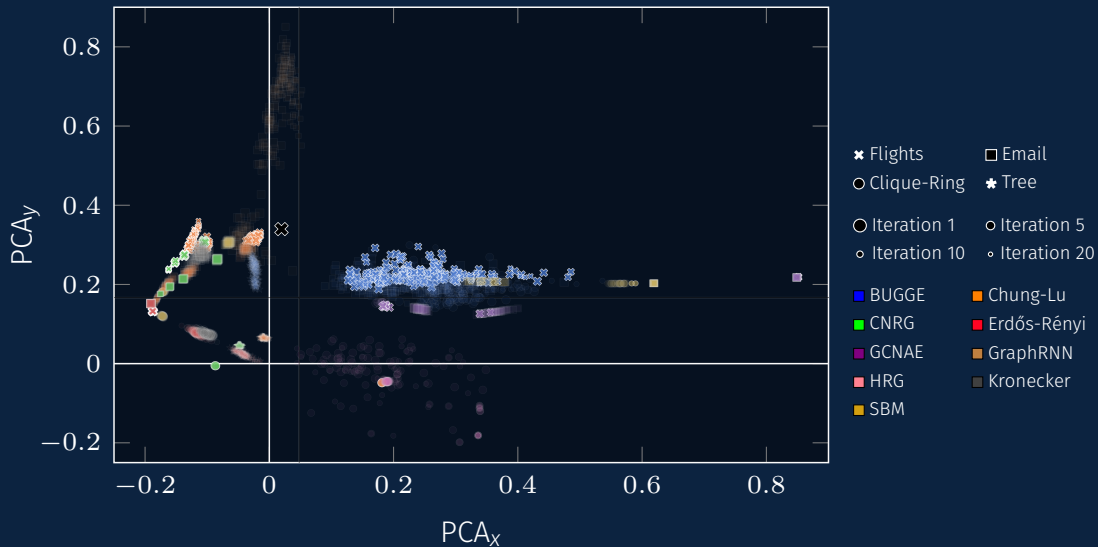
PORTRAIT DIVERGENCE OVER TIME



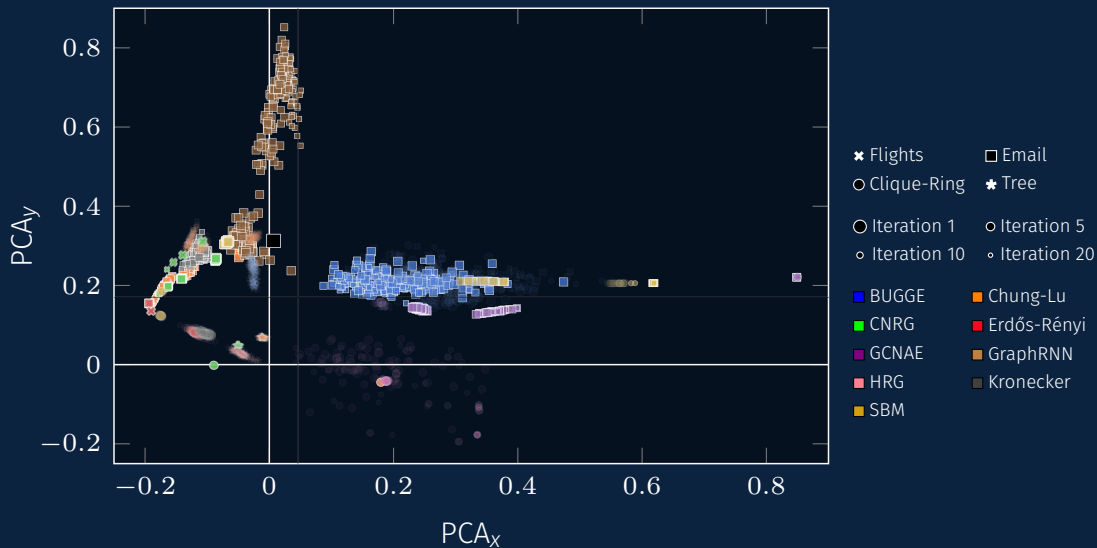
2D PCA ON GRAPHLET COUNT VECTORS



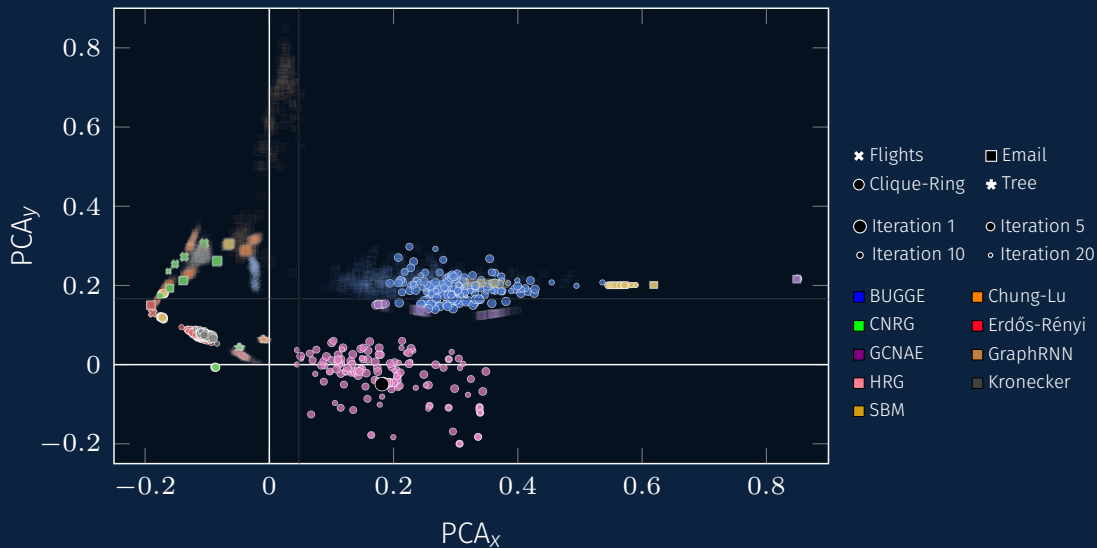
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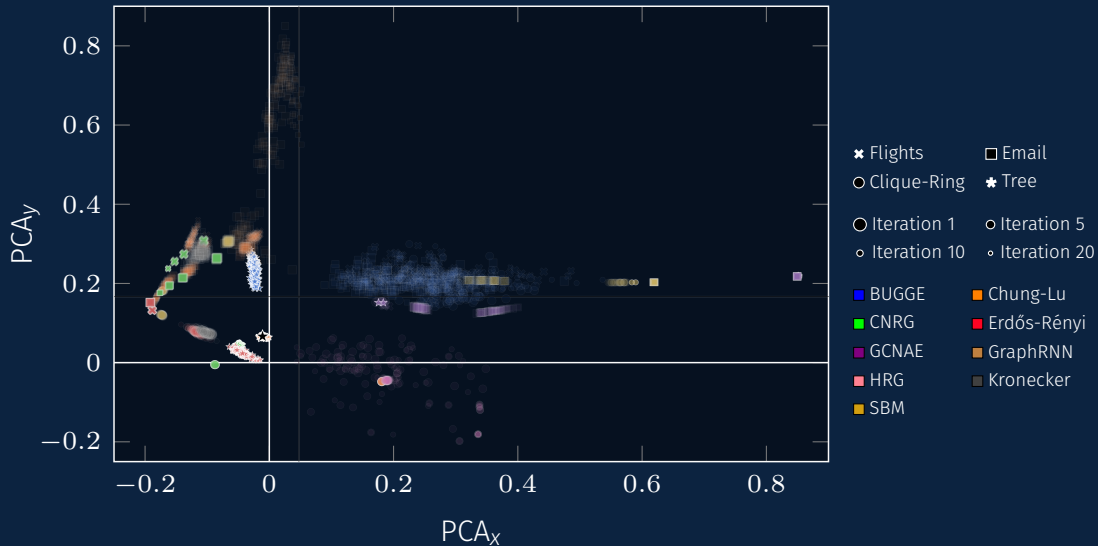
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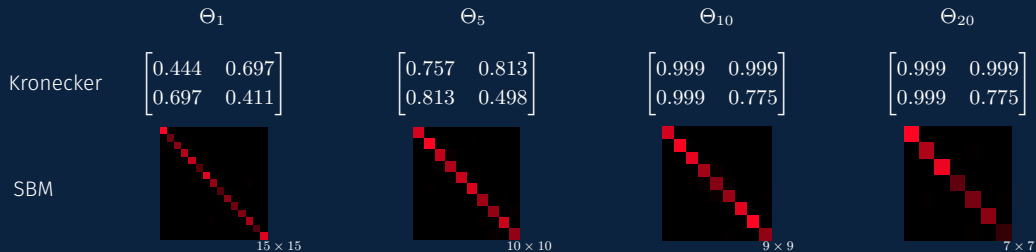
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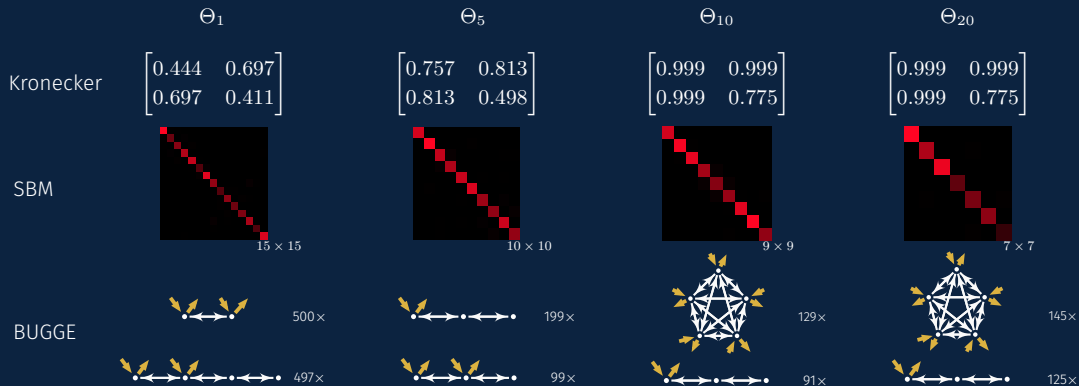
INVESTIGATING INDIVIDUAL MODEL PARAMETERS ON THE CLIQUE RING DATASET

	Θ_1	Θ_5	Θ_{10}	Θ_{20}
Kronecker	$\begin{bmatrix} 0.444 & 0.697 \\ 0.697 & 0.411 \end{bmatrix}$	$\begin{bmatrix} 0.757 & 0.813 \\ 0.813 & 0.498 \end{bmatrix}$	$\begin{bmatrix} 0.999 & 0.999 \\ 0.999 & 0.775 \end{bmatrix}$	$\begin{bmatrix} 0.999 & 0.999 \\ 0.999 & 0.775 \end{bmatrix}$

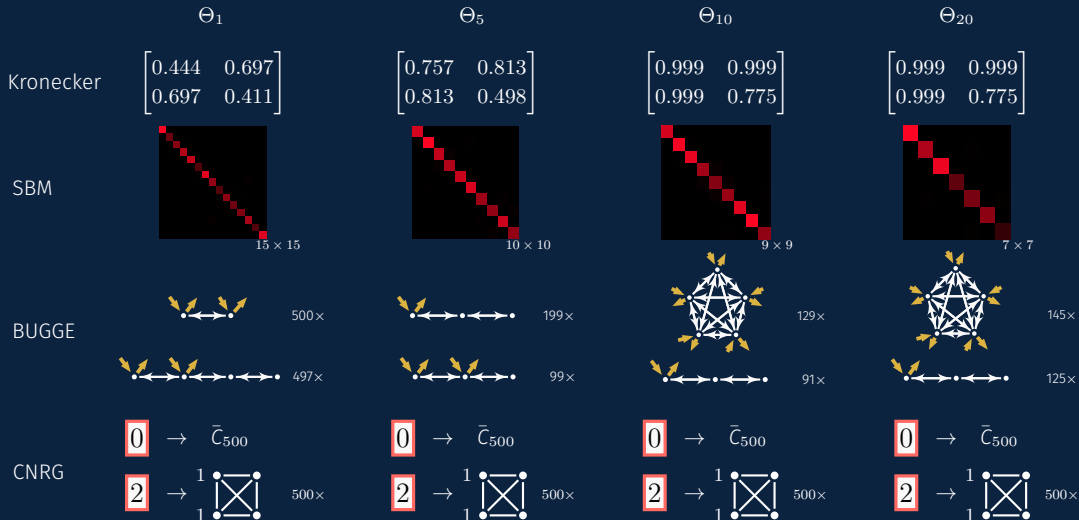
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CONCLUSIONS

Key Findings and Contributions

- Confirms previously known biases in Kronecker models.
- Uncovers distortion patterns in popularly used graph models
- Could be used as a tool to design better, more parsimonious graph models



Thank You!