

*Problem Set 4**Discrete Mathematics**Due on the 20th of February, 2024*(20 pts) 1.(a) Show $\forall x(\emptyset \subseteq x)$.(b) Show $\forall x(x \subseteq x)$.(c) Show $\forall x(\emptyset \in \mathbb{P}(x))$.(d) Show $\forall x(x \in \mathbb{P}(x))$.(e) Show $\forall x \forall y \forall z ((x \subseteq y) \wedge (y \subseteq z)) \Rightarrow x \subseteq z$.(10 pts) 2. We define the *intersection* and *difference* of any two sets x and y below.

$$x \cap y := \{z \mid z \in x \wedge z \in y\}$$

$$x \setminus y := \{z \mid z \in x \wedge z \notin y\}$$

(a) Show $\forall x \forall y \exists z(z = x \cap y)$.(b) Show $\forall x \forall y \exists z(z = x \setminus y)$.(20 pts) 3. We define the *union* of any two sets x and y below.

$$x \cup y := \{z \mid z \in x \vee z \in y\}$$

(a) Show $\forall x \forall y(x \cap y \subseteq x)$.(b) Show $\forall x \forall y(x \subseteq x \cup y)$.(c) Show $\forall x \forall y(\mathbb{P}(x) \cup \mathbb{P}(y) \subseteq \mathbb{P}(x \cup y))$.(d) Show $\forall x \forall y(x \cap y = x \Leftrightarrow x \in \mathbb{P}(y))$.(50 pts) 4. We define the *union over x* and *intersection over x* for any set x below.

$$\cup x := \left\{ z \mid \exists y(y \in x \wedge z \in y) \right\}$$

$$\cap x := \left\{ z \mid \forall y(y \in x \Rightarrow z \in y) \right\}$$

(a) Show that $\forall x(\cup \mathbb{P}(x) = x)$.(b) What is $\cup \emptyset$? Justify your answer with a proof.(c) What is $\cap \emptyset$? Justify your answer with a proof.(d) Is $\emptyset = \{z \mid z \in \emptyset\}$? Justify your answer with a proof.(e) Is $\emptyset = \{z \mid z \notin \emptyset\}$? Justify your answer with a proof.